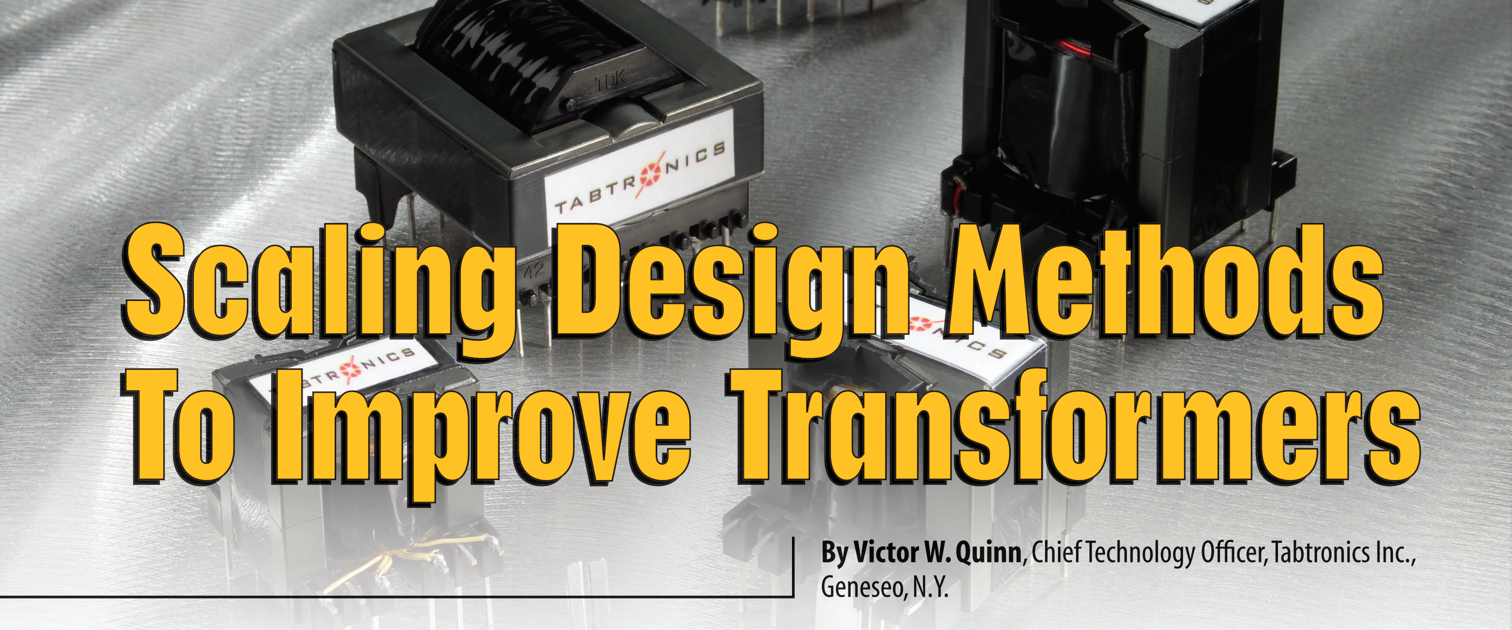

Scaling Design Methods To Improve Transformers

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Intuitive design techniques based on normalized loss and energy densities demonstrate real benefits for optimizing transformer configurations.

The size, performance and cost of a power electronic system are closely linked to the like parameters of the magnetic components. Since custom power transformers are widely used, transformer design is a frequent design process that has a significant impact on system performance. Yet most high-frequency transformers use less than 25% of the available core window for efficient current conduction. Two factors cause this relatively poor utilization: insulation requirements and eddy current losses.

Insulation requirements generally limit the total winding conductor cross-sectional area to less than half of the available core window. Induced high-frequency eddy currents often increase the apparent winding resistances by 50% or more. Therefore, less than 25% of the available core window is used effectively (**Fig. 1**). Interleaved windings, multifilar conductors, Litz wire and other approaches can decrease eddy current effects. However, these techniques add additional insulation penalties, which further reduce the net cross-sectional window area available for the winding conductors.

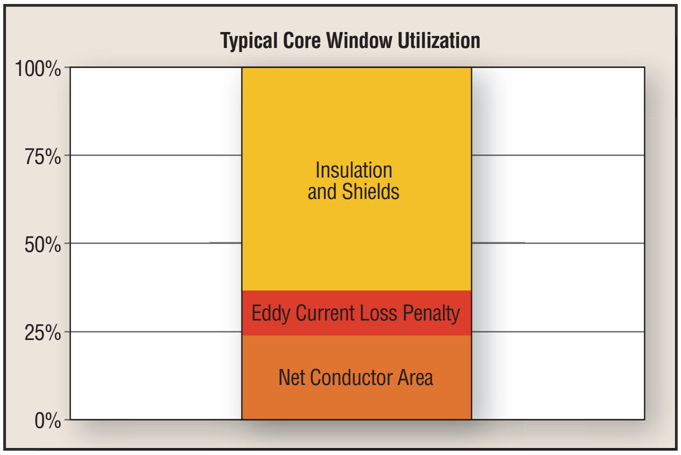

Fig. 1. *Typical core window utilization in a high-frequency transformer.*

Despite much published literature concerning the evaluation of transformer loss and energy at high frequencies, continued widespread interest in transformer design methods indicates that many designers seek more intuitive techniques to evaluate tradeoffs and assure an optimal configuration. This interest is understandable because many methods lead the designer to make repeated trial configurations until obtaining a satisfactory result. While Finite Element Analysis (FEA) and commercial software facilitate the evaluation of many trial configurations, the designer's imagination of improved configurations is fueled by understanding and insight.

A transformer design approach using normalized loss and energy densities inspires concepts of equivalent winding thicknesses that intuitively display optimal conductor and interleaving configurations.

Low-Frequency Transformer Design

Classical transformer design methods evaluate the potential volt-ampere (VA) capacity of a selected core in comparison to the VA requirement of a given power supply application. The VA capacity of a selected core is fundamentally limited by a consideration of losses with respect to temperature rise or efficiency requirements. At low frequencies, currents within conductors are largely uniform throughout the conductors' cross-sections. Therefore, dissipation for a given winding is readily calculated using the material resistivity and the geometric parameters of length and cross-section for the conductor. At low frequencies, for coils with multiple windings having uniform mean lengths of turn, minimum total loss is achieved by selecting appropriate conductors for each winding to yield uniform current density throughout the coil. For uniform resistivity, mean length of turn and current density, the low-frequency dissipation of a coil is given by:

$$\text{Low-frequency coil loss} = \rho \times \text{Cond}_{VOL} \times J^2$$

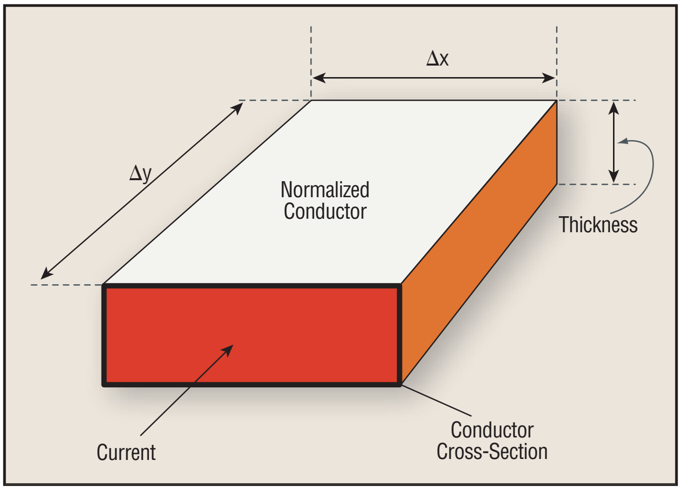

Fig. 2. *Depiction of a normalized conductor.*

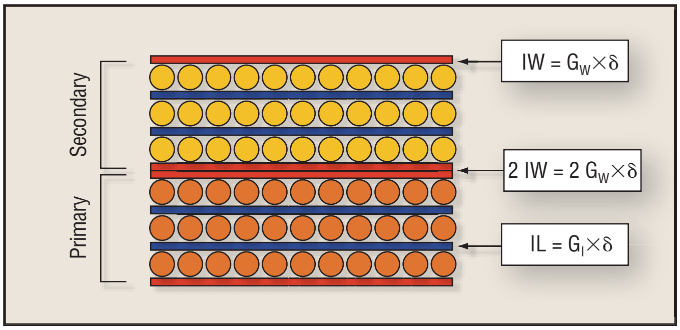

Fig. 3. *Cross-sectional view of winding with definitions of insulation thickness.*

where ρ is the conductor resistivity, $Cond_{VOL}$ denotes the effective total conductor volume, and J is the winding RMS current density.

Low-frequency loss can be rewritten using the concept of normalized resistance:

$$\text{Low-frequency coil loss} = \Omega\left[\sum_i NI_i\right]^2$$

where Ω is the resistance of a theoretically equivalent single-turn winding yielding the specified total $Cond_{VOL}$, and NI_i represents the RMS amp-turn product for the i^{th} winding in the coil.

The single-turn resistance Ω is given by the expression:

$$\Omega = \frac{\rho \times \Delta y}{\Delta x \times \text{Thickness}}$$

where Δy is the effective conductor length (or mean length of turn), Δx is the conductor width, and Thickness is the conductor thickness (**Fig. 2**). The denominator of this expression is equivalent to the total conductor cross-sectional area.

Therefore, low-frequency coil loss can be readily estimated using the normalized single-turn resistance Ω and the square of the sum of the RMS amp-turn products for all windings in the coil. The accuracy of this loss estimate depends on the accuracy of the estimate for the total conductor cross-sectional area. Insulation considerations generally limit the conductor area to less than half of the available core window area. The dominant insulations are those that isolate windings from core (bobbin), windings from other windings (interwinding) and layers from other layers (interlayer). The conductor thickness of the normalized winding can be estimated by reducing the available core window height by the anticipated bobbin, interwinding and interlayer thicknesses.

This approach may be extended to low-frequency magnetic energy in the coil. For a normalized, nonmagnetic, single-portion primary conductor, having uniform mean length of turn and uniform current density, the low-frequency magnetic energy is:

Normalized low-frequency magnetic energy =

$$\frac{\mu_0}{6}\frac{\Delta y}{\Delta x}\text{Thickness}$$

where μ_0 is the permeability constant for free space.

While coil insulation increases loss by decreasing the effective thickness of the conductor for the normalized winding, coil insulation increases magnetic energy in the portion by increasing the effective thickness of coil regions subject to magnetic-field penetration:

Normalized low-frequency magnetic energy =

$$\frac{\mu_0}{6}\frac{\Delta y}{\Delta x}\left[\text{Thickness}_{CU} + \frac{IL}{2}\frac{(2n_L - 1)(n_L - 1)}{n_L} + 3IW\right]$$

Insulation considerations generally limit the conductor area to less than half of the available core window area.

where Thickness_{CU} denotes the thickness of the accumulated copper conductors, n_L is the number of conductor layers in the winding portion, IL is the interlayer insulation thickness, and IW is the interwinding insulation thickness (**Fig. 3**). (Note that total interwinding insulation thickness is 2IW since each portion is taken to contribute half of the total interwinding insulation thickness.) Following the result for normalized resistance, low-frequency magnetic energy for a selected winding portion can be estimated using normalized low-frequency magnetic energy ($LFME_{NORM}$) with the square of the sum of the RMS amp-turn winding product in the portion:

Low-frequency magnetic energy =

$$LFME_{NORM}\left[\sum_{PORTION} NI_i\right]^2$$

For a specific application and a transformer having given conductor and core regions, the normalized expression for low-frequency winding loss can be combined with an empirically derived core loss result to yield:

Total low-frequency transformer loss =

$$F_{COIL} \times N_P{}^2 + \frac{F_{CORE}}{N_P^n}$$

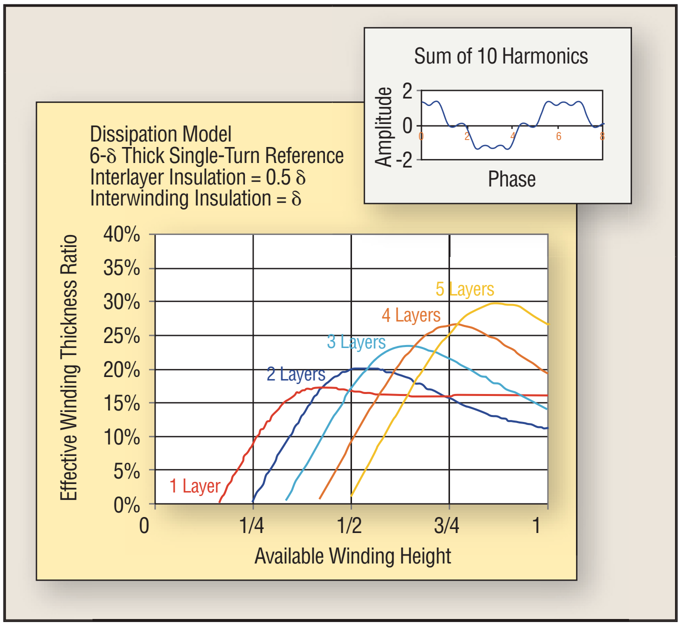

Fig. 4. *Loss-equivalent thickness for a bridge transformer primary.*

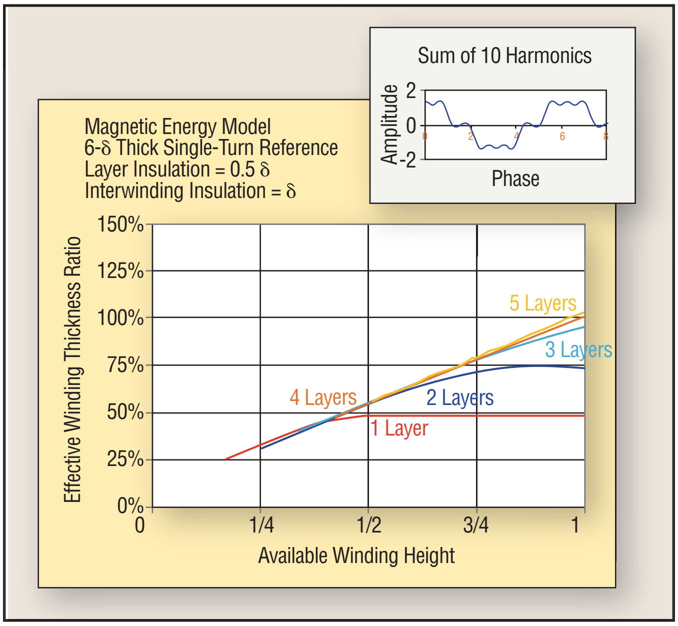

Fig. 5. *Energy-equivalent thickness for a bridge transformer primary.*

where F_{COIL} and F_{CORE} are parameters relating exponential functions of primary turns (N_P) to coil and core losses, respectively, and n is the empirically derived exponential variation of core loss with magnetic flux density. Without a constraint of core saturation, the value of primary turns that minimizes this expression for total low-frequency transformer loss is given by:

$$N_P = \sqrt[n+2]{\frac{n \times F_{CORE}}{2 \times F_{COIL}}}$$

Therefore, low-frequency transformer design involves normalized empirical core loss data and estimation of effective total conductor volume considering insulation, clearances, electrostatic shields and other coil cross-sectional area penalties.

High-Frequency Transformer Design

High-frequency power electronic applications cause added transformer design considerations. Coil loss is significantly affected by induced eddy currents within the conductors as a result of magnetic-field intensities. Further, high-frequency excitation and circuit topology requirements generate constraints on leakage inductance (stray magnetic energy). The inherent complex waveshapes necessitate consideration of dc and higher-order harmonic components.

For a single-conductor layer, having one conductor surface at zero magnetic-field intensity, high-frequency conductor loss and magnetic-energy storage can be calculated using the concept of dimensionless density functions[1]:

Conductor layer power =

$$p_0(R = 0, B) \times \rho \frac{\Delta y}{\Delta x \times \delta}(NI)^2$$

Conductor layer magnetic energy =

$$e_0(R = 0, B) \times \mu_0 \frac{\Delta y}{\Delta x}\delta(NI)^2$$

where δ is the skin depth for the conductor material at the specified frequency, Δx represents the width of the conductor layer, Δy represents the mean length of turn of the conductor layer, R is the ratio of sinusoidal surface magnetic-field intensities, B is the ratio of conductor thickness to skin depth, and p_0 and e_0 are the dimensionless density functions for loss and magnetic energy, respectively.

Although p_0 and e_0 are functions involving complex terms, the resultant expressions are similar to the low-frequency expressions presented earlier because they largely depend on simple geometric factors and the square of the amp-turn product. As a mathematical approach for calculation of dissipation, effective conductor thickness can be taken as the skin depth divided by the loss-density function. The loss-density function adjusts the effective thickness as a result of induced magnetic-field and current distributions within the conductor layer. In truth, conductor thickness is not changing with excitation. The theoretical convenience of equivalent thickness is merely a simplifying calculation method to evaluate net dissipation and stored energy effects in a conductor layer. Effective thickness represents the effects of complex current density and magnetic-field distributions, which vary greatly within the conductor cross-sections. Therefore, the dimensionless density functions yield intuitive calculations of loss and magnetic energy using the equivalent low-frequency expressions. Further derivations will be required to consider the general case for a multiple-layer winding undergoing complex excitation.

Nonsinusoidal Currents

Transformer winding current may contain a bias component, and in this event, the winding current can be represented by the harmonic decomposition:

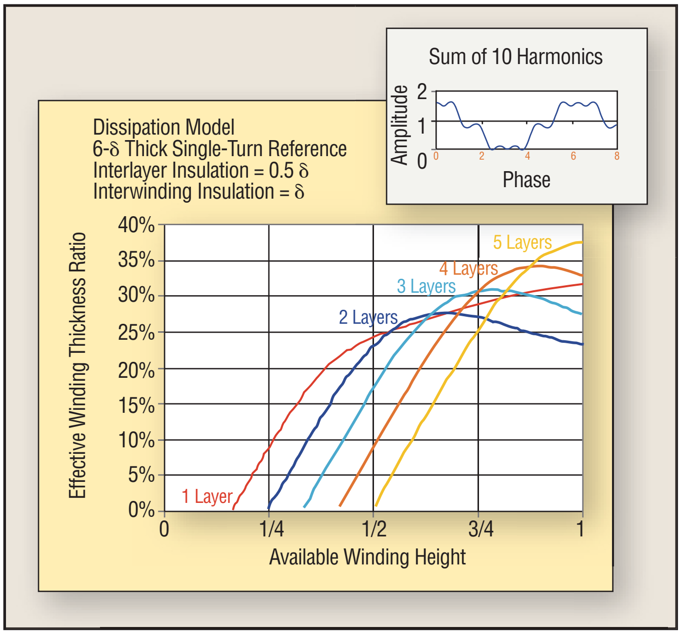

Fig. 6. *Loss-equivalent thickness for a bridge transformer secondary.*

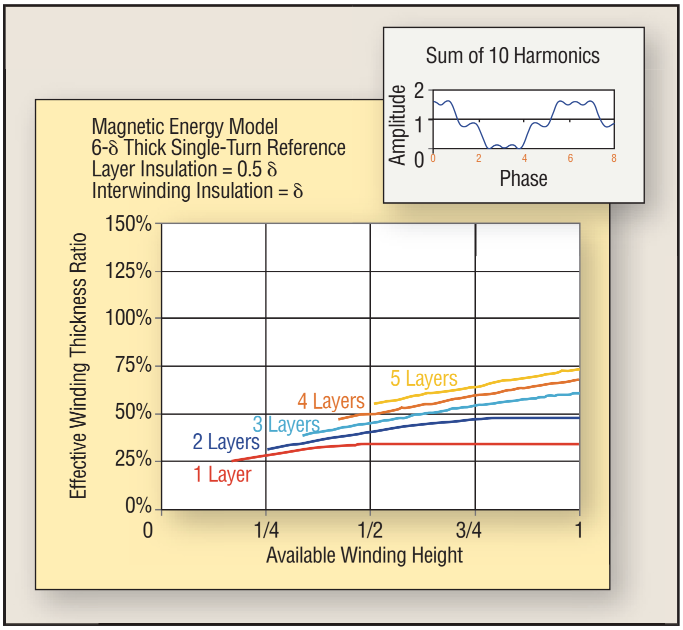

Fig. 7. *Energy-equivalent thickness for a bridge transformer secondary.*

$$I_{DC} = \alpha_0 \times I_{RMS}$$
$$I_{RMS1} = \alpha_1 \times I_{RMS}$$
$$I_{RMSk} = \alpha_K \times I_{RMS}$$

where I_{RMSk} is the RMS current value at the k[th] harmonic, I_{RMS} is the RMS value of the complex waveshape, and α_K is the ratio of the RMS value of the k[th] harmonic to the RMS value of the complex waveshape.

The equivalent loss-density function for a given layer is then given by:

$$p_{0equiv}(R, B_1) = \alpha_0^{\,2}\frac{(1-R)^2}{B_1} + \sum_i \sqrt{i}\ \alpha_0^{\,2} p_0(R, \sqrt{i} \times B_1)$$

where B_1 is the ratio of conductor thickness to skin depth for the first harmonic.

Similarly, the equivalent ac magnetic-energy density function for a given layer is:

$$e_{0equiv}(R, B_1) = \sum_i \frac{\alpha_i^{\,2}}{\sqrt{i}} e_0(R, \sqrt{i} \times B_1)$$

These equivalent density functions can be used to derive expressions for loss- and energy-equivalent conductor thicknesses:

Loss-equivalent thickness =

$$\frac{\delta}{\sum_{n=1}^{n_L}\left(\frac{n}{n_L}\right)^2 p_{0equiv}\left(1-\frac{1}{n}, B_1\right)}$$

Magnetic-energy-equivalent thickness =

$$\delta\left[3 \times G_W + \frac{G_I}{2}\frac{(2n_L-1)(n_L-1)}{n_L} + 6\sum_{n=1}^{n_L}\left(\frac{n}{n_L}\right)^2 e_{0equiv}\left(1-\frac{1}{n}, B_1\right)\right]$$

where n_l is the number of layers, $G_I \times \delta$ is the interlayer insulation thickness and $G_W \times \delta$ is the interwinding insulation thickness for the selected winding portion.

Although the expressions for loss and magnetic energy invoke complex density terms that appear formidable, let's turn our attention to simple graphical methods using concepts of effective normalized winding thickness and normalized winding height. These concepts are readily accomplished using common computer methods and facilitate graphical recognition of optimal coil designs considering insulation and eddy current penalties.

Equivalent Thickness Methods

As described earlier, the transformer design process involves the calculation of coil loss in consideration of a VA requirement arising from circuit application conditions. For low frequencies, the total coil loss can be evaluated using the product of the single-turn resistance of a volume-equivalent single-turn conductor and the square of the amp-turn product through the conductor cross-section. We will seek a similar intuitive calculation method for loss and magnetic energy in a winding portion undergoing complex excitation.

To this end, we will develop a graphical relationship between the effective winding thicknesses and the physical utilization of an available winding height. This graphical approach facilitates the selection of optimal layers and winding interleaves, including the effects of layer-insulation thickness and considering constraints of maximum leakage-inductance energy and maximum coil height. Loss and magnetic energy can then be evaluated following the expressions for low frequency applied to a normalized conductor having the appropriate equivalent thickness.

This method is illustrated in **Figs. 4** through **9** as applied to a winding portion with maximum overall winding thickness of six skin depths, five conductor layers maximum, and

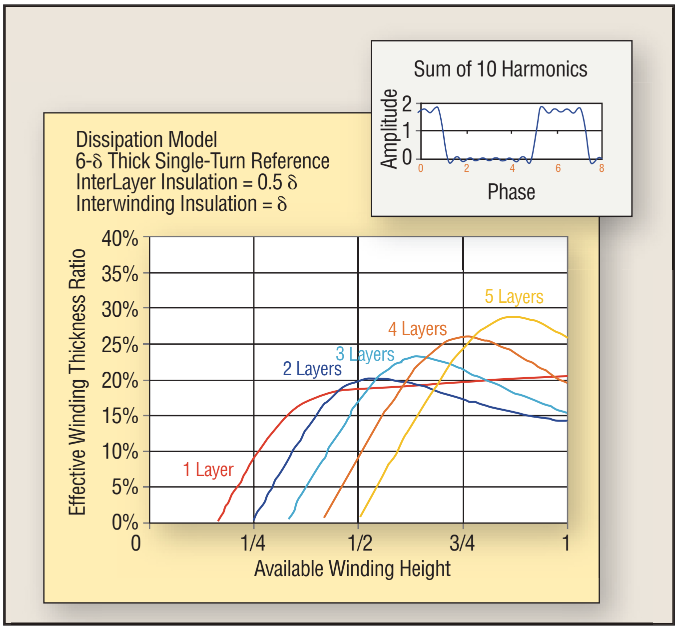

Fig. 8. *Loss-equivalent thickness for a forward transformer primary.*

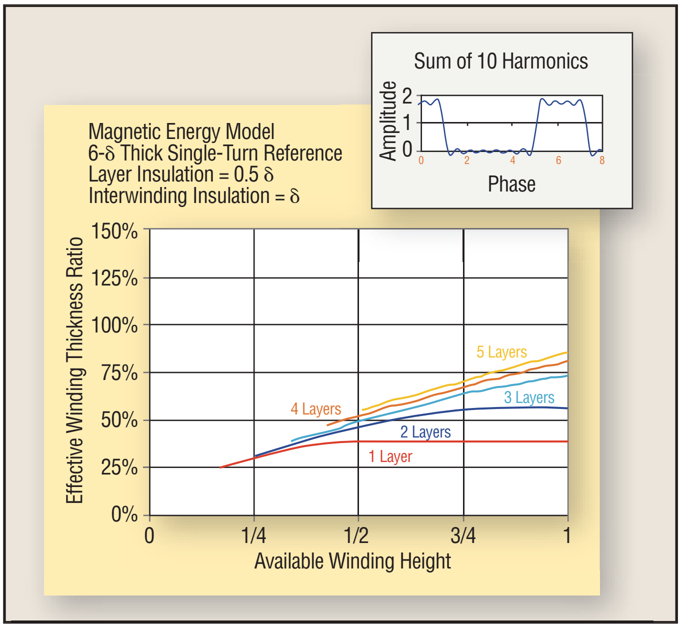

Fig. 9. *Energy-equivalent thickness for a forward transformer primary.*

interlayer- and interwinding-insulation thicknesses of half a skin depth each. **Figs. 4** and **5** pertain to the primary winding of a transformer used in a bridge configuration operating at 62.5% duty cycle. The current was normalized to yield unity RMS value.

Fig. 4 shows that maximum loss-equivalent-winding thickness is achieved when 88% of the winding height is used with five conductor layers. For this configuration, the loss-equivalent-winding thickness is 28% of the available winding height. However, as a result of

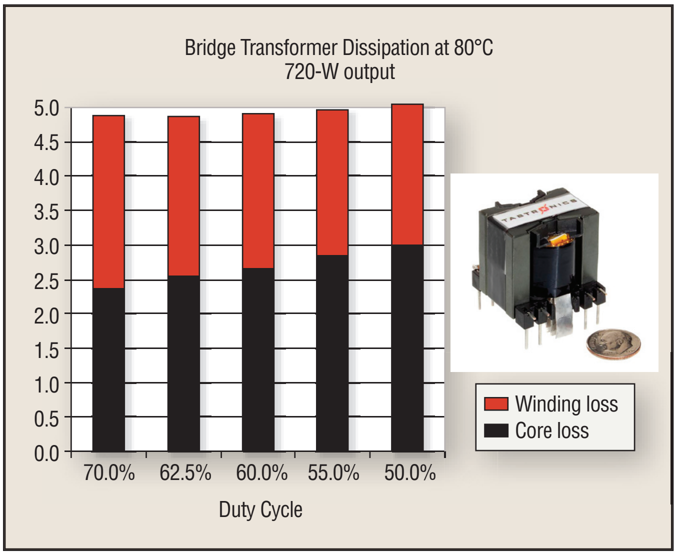

Fig. 10. *Measured total loss of 150-kHz bridge transformer providing 12 V at 60 A.*

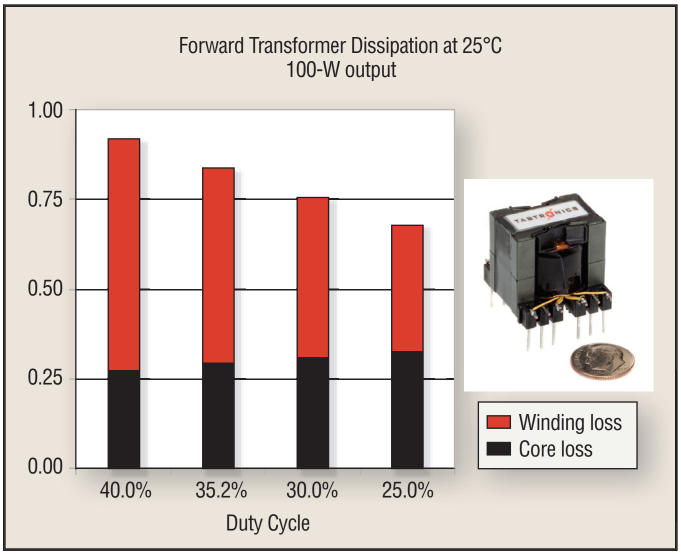

Fig. 11. *Measured total loss of 100-kHz forward transformer providing 20 V at 5 A.*

magnetic energy stored in insulation regions, the corresponding magnetic-energy-equivalent-winding thickness is 92% of the available winding height (**Fig. 5**). Interleaving effects can be readily evaluated by considering constructions that fit within a fraction of the available winding height. For example, a simple interleave of two portions can be examined by considering winding configurations that fit within half of the available winding height.

In **Fig. 4**, when winding height is limited to 50%, two layers provide a maximum equivalent thickness of 20%. However, two portions can be used with an interleave strategy to double the equivalent winding thickness and in this case achieve 40% of the available winding height. Therefore, an interleaved configuration that splits the winding into two portions of two layers each dissipates 30% less power than the optimal single-winding-portion configuration. This analysis can be readily extended to other interleave strategies by considering the constraint on available winding height to be 1/n, where n is the number of interleaved winding portions. Resultant-loss and energy-equivalent thicknesses will respectively increase and decrease by a factor of n with appropriate interleaving.

When an optimal winding strategy is selected, the resultant-loss and magnetic-energy-equivalent winding thicknesses can be used to determine the normalized dissipation and energy storage for equivalent single-turn conductors. These results are readily scaled for the specific application using the square of the amp-turn product through the cross-section.

The secondary current of the bridge transformer contains a dc bias component because rectifier elements force unidirectional currents. **Figs. 6** and **7** display the results for one of the half secondaries again operating at 62.5% duty cycle. The significant dc component causes thicker conductors to have greater benefit; therefore, two portions of a single layer each provide the lowest winding dissipation with a combined loss-equivalent thickness of 48%. **Fig. 7** indicates that the lowest ac magnetic energy is also achieved with two single-layer portions, yielding a total magnetic-energy-equivalent thickness of 17%. Note that this equivalent thickness is correlated to the RMS current of the total waveshape, including the dc bias component.

The forward-converter application also generates a significant bias component in the current waveshape. For a duty cycle of 35.2%, **Fig. 8** indicates that two portions of two layers each are the preferred choice to minimize loss with a combined loss-equivalent thickness of 40%. The harmonic distribution of the forward converter causes thick single-layer portions to be less beneficial. Clearly, optimal transformer design depends on an accurate definition of application waveshapes and constituent harmonic components.

Transformer Examples

Two transformers were designed using the methods outlined earlier. **Fig. 10** summarizes total transformer dissipation at 80°C as a function of duty cycle for a bridge transformer operating at 150 kHz and providing 12 V at 60 A. **Fig. 11** displays the results at 25°C for a forward transformer operating at 100 kHz and providing 20 V at 5 A.

Open-circuit test methods were used to measure core loss at each of the harmonic components of application voltage. Short-circuit test methods were used to measure ac resistance and evaluate ac winding loss at each of the harmonic components of application current. DCR test methods were used to evaluate dc loss of windings conducting bias currents. **Figs. 10** and **11** summarize total dissipation using these comprehensive methods. Both transformers achieve efficiencies in excess of 99%. **PETech**

References

1. Quinn, V. "New Technology Reduces Eddy Current Dissipation in Windings," Properties and Applications of Magnetic Materials Conference Record, 2005.